# 4.1 Exponential Functions and Graphs  MATH 1610  THOMPSON

1) Evaluate the exponential expression.

$(-4)^2$     Make sure to put $(-4)^2$ is calculator = 16

2) Simplify the given expression.

$9^{-2}$     $= \dfrac{1}{81}$     calculator: 9^-2 then 2nd PRB

3) Simplify the given expression.

$7^{-4} \cdot 7^3$     7^ - 4 times 7^3 2nd PRB

$7^{-4} \cdot 7^3 = \dfrac{1}{7}$  (Type an integer or a simplified fraction.)

4) Simplify the expression.

$\left(4^{-1}\right)^{-3}$

$\left(4^{-1}\right)^{-3} = $ 64  (Type an integer or a simplified fraction.)

5) Simplify the given expression.

$\sqrt{49}$

$\sqrt{49} = $ 7

6) Simplify the given expression.

$\sqrt{(-8)^2}$

$\sqrt{(-8)^2} = $ 8

**7)** Find the value of the given expression if $x = 7$ and $y = -3$.

$$5xy^{-1}$$

$$\frac{5(7)}{-3}$$ negative exponent makes a fraction

$5xy^{-1} = \boxed{-\dfrac{35}{3}}$ (Simplify your answer.)

**8)** Find the value of the expression if $x = 3$ and $y = -4$.

$$x^2 + y^2$$

$$\frac{(3)^2 + (-4)^2}{}$$

$x^2 + y^2 = \boxed{25}$ (Simplify your answer.)

**9)** Find the value of the expression if $x = 4$ and $y = -1$.

$$(xy)^2$$

$$\frac{(4 \cdot (-1))^2}{}$$

$(xy)^2 = \boxed{16}$ (Simplify your answer.)

**10)** Find the value of the expression if $x = 3$.

$$\sqrt{x^2}$$

$$\frac{\sqrt{(-3)^2}}{}$$

$\sqrt{x^2} = \boxed{3}$ (Simplify your answer.)

**11)** Find the value of the expression if $x = 6$ and $y = -7$.

$$\sqrt{x^2 + y^2}$$

$$\frac{\sqrt{6^2 + (-7)^2}}{}$$

$\sqrt{x^2 + y^2} = \boxed{\sqrt{85}}$

**12)** Find the value of the expression if $x = 8$ and $y = -1$.

$$x^y \qquad \text{Negative exponent makes a fraction}$$

$x^y = \dfrac{1}{8}$ (Simplify your answer.)

**13)** $4^2 = 16$ (Type an integer or a simplified fraction.)

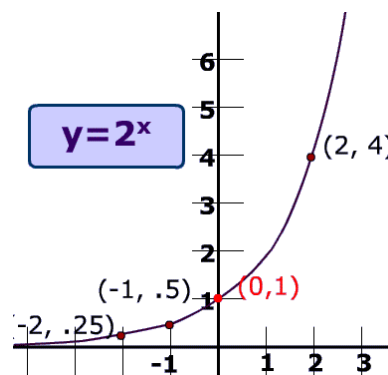$8^{\frac{2}{3}} = 4$ (Type an integer or a simplified fraction.)

$3^{-3} = \dfrac{1}{27}$ (Type an integer or a simplified fraction.)

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The exponential function f(x) = a , a>0, a≠1, has <u>no x-intercept</u> because it is above the x-axis.

**Exponential Graph →**    $y=2^x$

Domain: (-∞,∞)

Range:   (0,∞)

(2, 4)

(-1, .5)  (0,1)

-2, .25)

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**14)** Decide whether the following statement is true or false.

The domain of the exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$, is the set of all real numbers.

Choose the correct answer below.

○ False

✔ True

**15)** Decide whether the following statement is true or false.

The graph of the exponential function $f(x) = a^x$, $a > 0$, $a \neq 1$, has no x-intercept.

Choose the correct answer below.

✔ True

○ False

**16)** Which of the following is the range of the exponential function $f(x) = a^x$, $a > 0$ and $a \neq 1$?

Choose the correct answer below.

- $(-\infty, \infty)$
- ✔ $(0, \infty)$
- $(-\infty, 0)$
- $(-\infty, 0) \cup (0, \infty)$

**17)  Determine the function is linear, exponential, or neither?**

**To check linear:** check slope on two set of points

$$\frac{1-11}{0-1} = \frac{-10}{-1} = 10$$

$$\frac{11-121}{1-2} = \frac{-110}{-1} = 110 \text{ no}$$

| X | f(x) |
|---|------|
| -1 | 1/11 |
| 0 | 1 |
| 1 | 11 |
| 2 | 121 |
| 3 | 1331 |

**To check exponential:**

$$\frac{f(x+1)}{f(x)} = a \text{ where } f(x) = Na^x \text{ and } f(x) = N$$

$$\frac{f(0)}{f(-1)} = \frac{1}{1/11} = 11$$

$$\frac{f(1)}{f(0)} = \frac{11}{1} = 11 \quad \text{yes, therefore: f(0) = 1 then } \boxed{f(x) = 11^x}$$

*HINT when first is $(-1, \frac{1}{n})$ and second is (0,1) then

f(x) = n^x

**18) Determine the function is linear, exponential, or neither?**

**To check linear:** check slope on two set of points

| X | f(x) |
|---|------|
| -1 | 13/12 |
| 0 | 13 |
| 1 | 156 |
| 2 | 1872 |
| 3 | 22464 |

$$\frac{13-156}{0-1} = \frac{-143}{-1} = 143$$

$$\frac{156-1872}{1-2} = \frac{-1716}{-1} = 1716 \text{ no}$$

**To check exponential:**

$$\frac{f(x+1)}{f(x)} = a \text{ where } f(x) = Na^x \text{ and } f(x) = N$$

$$\frac{f(0)}{f(-1)} = \frac{13}{13/12} = 12$$

$$\frac{f(1)}{f(0)} = \frac{165}{13} = 12 \quad \text{yes, therefore: f(0) = 13 then } \boxed{f(x)=13 \cdot 12^x}$$

*HINT when first is $(-1, \frac{c}{n})$ and second is (0,c) then

$$f(x) = c \cdot n^x$$

**19)**

Determine whether the function given by the table is linear, exponential, or neither. If the function is linear, find a linear function that models the data; if it is exponential, find an exponential function that models the data.

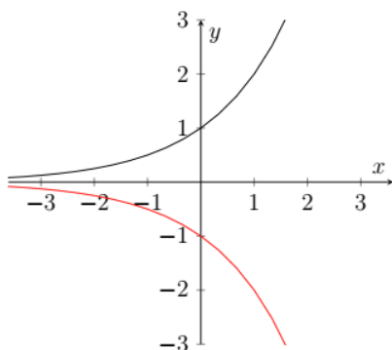| x | f(x) |
|---|------|
| −1 | 2 |
| 0 | 5 | y-intercept |
| 1 | 8 |
| 2 | 11 |
| 3 | 14 |

**To check linear:** check slope on two set of points

$$\frac{5-2}{0+1} = 3 \quad \text{slopes are the same}$$
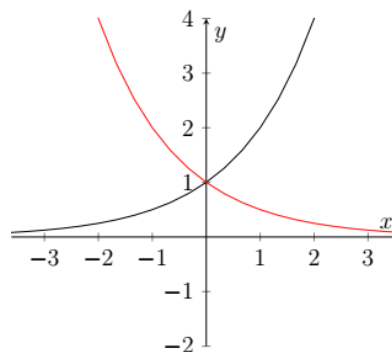
$$\frac{8-5}{1-0} = 3 \quad \text{linear} \quad y = 3x + 5$$

ORIGINAL GRAPH IN BLACK

REFLECT ACROSS X IN RED

20) Match the graph to one of the following functions.

A. $y = 7^x$

B. $y = 7^{-x}$

C. $y = -7^x$
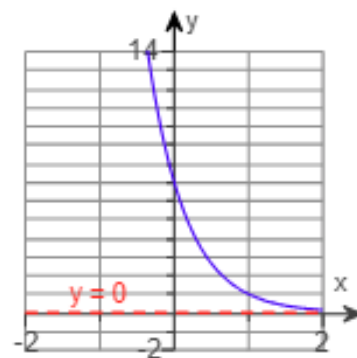
D. $y = -7^{-x}$

E. $y = 7^x - 1$

F. $y = 7^{x-1}$

G. $y = 7^{1-x}$

H. $y = 1 - 7^x$

$7^{-x+1}$

*Reflects across the y-axis and shifts right 1    G

21) Match the graph to one of the following functions.

A. $y = 7^x$

B. $y = 7^{-x}$
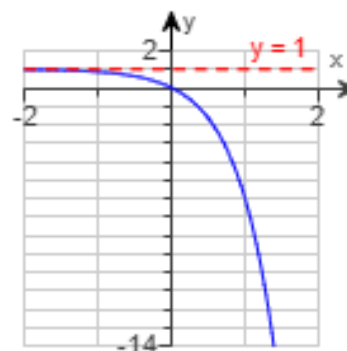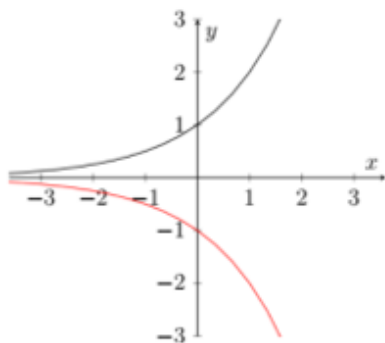
C. $y = -7^x$

D. $y = -7^{-x}$

E. $y = 7^x - 1$

F. $y = 7^{x-1}$

G. $y = 7^{1-x}$

H. $y = 1 - 7^x$

$-7^x + 1$

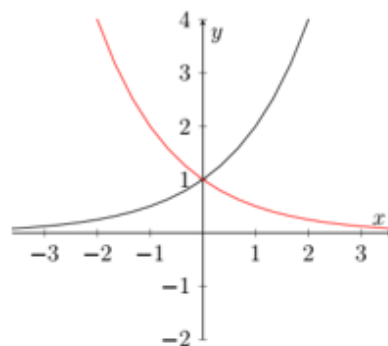*Reflects across the x-axis and shifts up 1     H
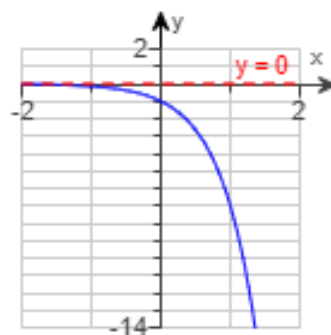
ORIGINAL GRAPH IN BLACK

REFLECT ACROSS X IN RED

ORIGINAL GRAPH IN BLACK

REFLECT ACROSS X IN RED

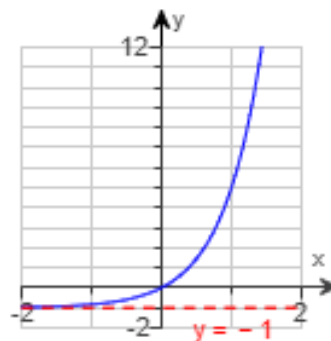**22)** Match the graph to one of the following functions.

A. $y = 7^x$ 　　　　 B. $y = 7^{-x}$

C. $y = -7^x$ 　　　 D. $y = -7^{-x}$

E. $y = 7^x - 1$ 　　 F. $y = 7^{x-1}$

G. $y = 7^{1-x}$ 　　 H. $y = 1 - 7^x$

*Reflects across the x-axis  C

**23)** Match the graph to one of the following functions.

A. $y = 6^x$ 　　　　 B. $y = 6^{-x}$

C. $y = -6^x$ 　　　 D. $y = -6^{-x}$

E. $y = 6^x - 1$ 　　 F. $y = 6^{x-1}$
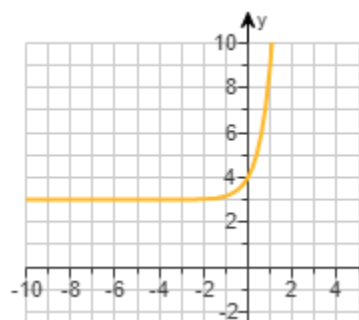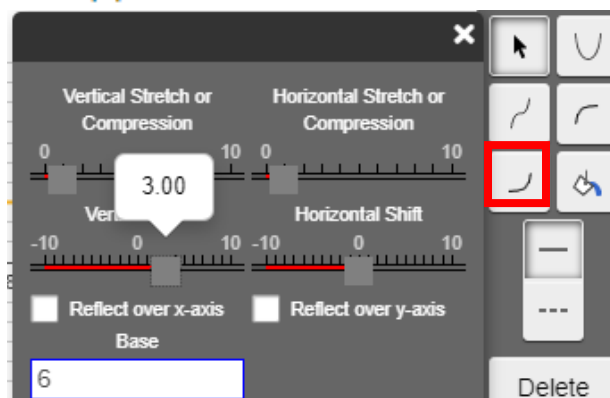
G. $y = 6^{1-x}$ 　　 H. $y = 1 - 6^x$

*Shifts down 1  E

**24)** Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$f(x) = 6^x + 3$   **shifts up 3**





What is the domain of $f(x) = 6^x + 3$?

$(-\infty, \infty)$

(Type your answer in interval notation.)

What is the range of $f(x) = 6^x + 3$?

$(3, \infty)$

**Vertical shift up 3**

The function has one horizontal asymptote, $y = 3$.   **Up 3**

(Type an equation.)

What is the y-intercept of $f(x) = 6^x + 3$?

4   (Type an integer or a simplified fraction.)

**Plug 0 in for x**

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**25)** Use transformations to graph the function. Determine its domain, range, and horizontal asymptote.

$f(x) = 5^x - 4$   **Down 4**

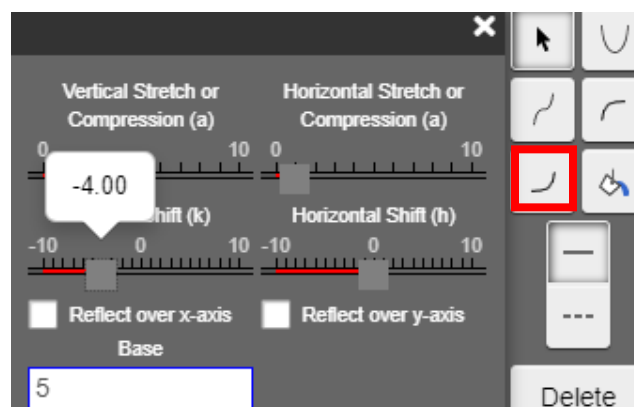What is the domain of $f(x) = 5^x - 4$?

$(-\infty, \infty)$

(Type your answer in interval notation.)

What is the range of $f(x) = 5^x - 4$?

$(-4, \infty)$

**Vertical down up 4**



What line is the horizontal asymptote of $f(x) = 5^x - 4$?   **Down 4**

$y = -4$   (Type an equation.)

**26)** Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

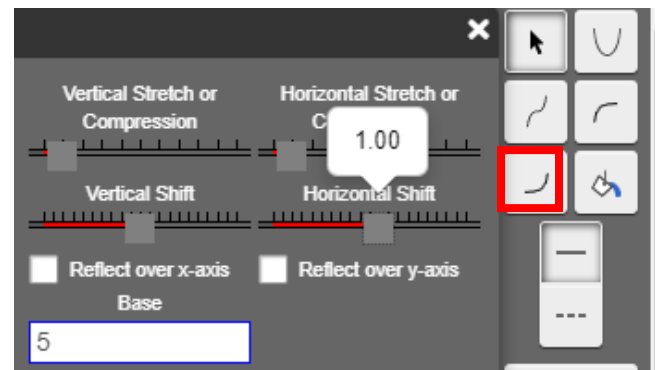$f(x) = 5^{x-1}$    Right 1

What is the domain of $f(x) = 5^{x-1}$?

$(-\infty, \infty)$
(Type your answer in interval notation.)

What is the range of $f(x) = 5^{x-1}$? **+ 0**

$(0, \infty)$
(Type your answer in interval notation.)

The function has one horizontal asymptote, $y = 0$.

What is the y-intercept of $f(x) = 5^{x-1}$?

$\dfrac{1}{5}$    Plug 0 in for x $5^{-1}$

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**27)** Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

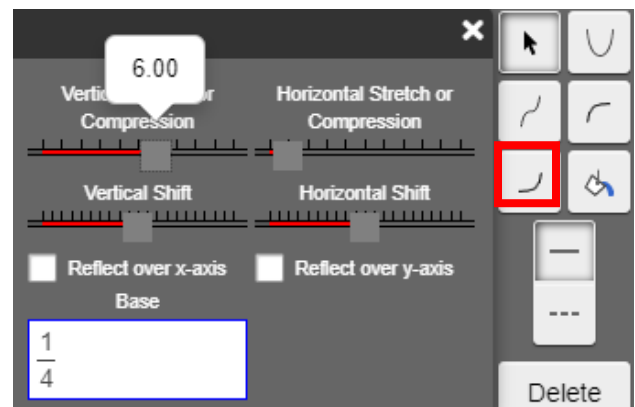$f(x) = 6 \cdot \left(\dfrac{1}{4}\right)^{x}$    Vertical stretch of 6

What is the domain of $f(x) = 6 \cdot \left(\dfrac{1}{4}\right)^{x}$?

$(-\infty, \infty)$
(Type your answer in interval notation.)

What is the range of $f(x) = 6 \cdot \left(\dfrac{1}{4}\right)^{x}$? **+ 0**

$(0, \infty)$

The function has one horizontal asymptote, $y = 0$.

What is the y-intercept of $f(x) = 6 \cdot \left(\dfrac{1}{4}\right)^{x}$?

6 (Type an integer or a simplified fraction.)

Plug 0 in for x

**28)** Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$f(x) = 3^{-x} - 2$  Reflects across y

down 2



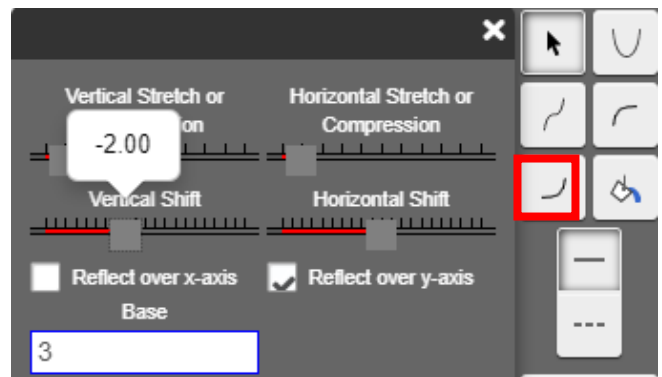What is the domain of $f(x) = 3^{-x} - 2$?

$(-\infty, \infty)$
(Type your answer in interval notation.)

What is the range of $f(x) = 3^{-x} - 2$?

$(-2, \infty)$
(Type your answer in interval notation.)

Vertical down up 2

The function has one horizontal asymptote, $y = -2$.  Down 2
(Type an equation.)

What is the y-intercept of $f(x) = 3^{-x} - 2$?

$-1$  (Type an integer or a simplified fraction.)  Plug 0 in for x

**29)** Use transformations to graph the function. Determine the domain, range, horizontal asymptote, and y-intercept of the function.

$f(x) = 1 + 3^{x-2}$  →  f(x)= 3$^{x-2}$ + 1

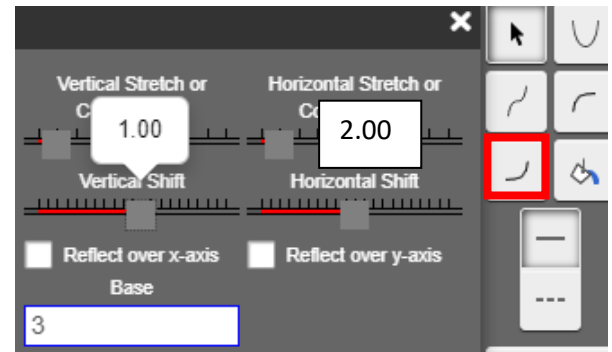Right 2, up 1



What is the domain of $f(x) = 1 + 3^{x-2}$?

$(-\infty, \infty)$
(Type your answer in interval notation.)

What is the range of $f(x) = 1 + 3^{x-2}$?

$(1, \infty)$

Vertical shift up 1

The function has one horizontal asymptote, $y = 1$.  Up 1
(Type an equation.)
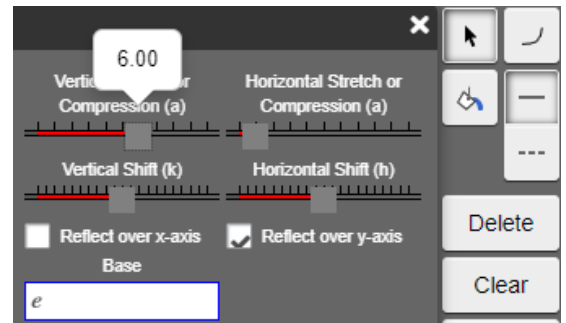
What is the y-intercept of $f(x) = 1 + 3^{x-2}$?

$\dfrac{10}{9}$  (Type an integer or a simplified fraction.)

$1 + 3^{-2} = 1 + \dfrac{1}{9}$

30) Begin with the graph of $y = e^x$ and use transformations to graph the following function. Determine the domain, range, and horizontal asymptote of the function.

$f(x) = 6 e^{-x}$

Reflects across the y

vertical stretch of 6

The domain of $f(x) = 6 e^{-x}$ is $(-\infty, \infty)$.
(Type your answer in interval notation.)

The range of $f(x) = 6 e^{-x}$ is $(0, \infty)$.
(Type your answer in interval notation.)

The horizontal asymptote of $f(x) = 6 e^{-x}$ is
$y = 0$.

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31) Solve $4^{-x} = 256$         get the same base

$4^{-x} = 4^4$

set exponents equal and solve         $-x = 4$     $x = -4$

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32) Solve $\left(\frac{3}{5}\right)^x = \frac{27}{125}$ get the same base

$\left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^3$

set exponents equal         $x = 3$ ✓

33) Solve $5^{4x+1} = 25$     get the same base

$5^{4x+1} = 5^2$

set exponents equal and solve     $4x + 1 = 2$

$4x = 1$     $x = \dfrac{1}{4}$

34) Solve $8^{x^2-5} = 64^{2x}$  get the same base

$8^{x^2-5} = 8^{4x}$

set exponents equal and solve $x^2 - 5 = 4x$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1)$     $x = -1, 5$

Ex: $X^3 \cdot x^5 = x^{3+5}$

35) Solve $49^x \cdot 7^{x^2} = 2401^2$  get the same base

$7^{2x} \cdot 7^{x^2} = 7^8$

set exponents equal and solve $2x + x^2 = 8$

$x^2 + 2x - 8 = 0$

$(x-2)(x+4) = 0$     $x = -4, 2$

36) Solve $e^x = e^{3x+18}$

set exponents equal and solve $x = 3x + 18$

$-2x = 18$     $x = -9$

37) Solve $e^{x^2} = e^{8x} \cdot \dfrac{1}{e^{15}}$ get the same base $\qquad$ $\boxed{\dfrac{1}{e^{15}} = e^{-15}}$

$$e^{x^2} = e^{8x} \cdot e^{-15}$$

set exponents equal and solve $x^2 = 8x-15$
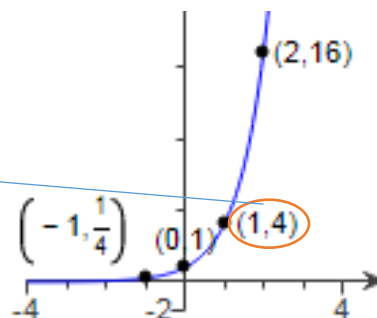
$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0 \qquad x = 3,5$$

38) If $6^x = 7$ then $6^{-2x} = ?$

$$7^{-2} = \dfrac{1}{49}$$

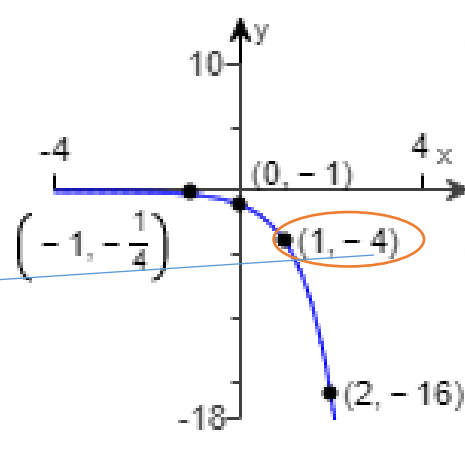39) Give the function of the graph:

$f(x) = 4^x$

$(1,4) \quad 4 = 4^1$


(2,16)
$\left(-1,\frac{1}{4}\right)$ (0,1) (1,4)
-4 -2 4

Determine the exponential function whose graph is given.

40)


10
-4 4 x
(0, -1)
$\left(-1,-\frac{1}{4}\right)$ (1, -4)
(2, -16)
-18

$f(x) = -4^x$

$(1,-4) \quad -4 = -4^1$

41) Suppose that $g(x) = 2^x + 2$

    a) What is g(-1)? When x = -1, what is the point on the
       graph? *Plug in -1 for x (in exponent)*

$$2^{-1} + 2 = \frac{5}{2} \quad \text{point (write as x,y coordinate)} \quad (-1, \frac{5}{2})$$

    b) If g(x) = 10, what is x? Give point on the graph.

$$10 = 2^x + 2 \quad \textit{Plug in 10 for g(x)}$$
$$8 = 2^x$$
$$2^3 = 2^x$$
$$x = 3 \quad \text{point (write as x,y coordinate)} \quad (3,10)$$

42) If a single pane of glass obliterates 9% of the light passing through it,
then the percent p of the light that passes through n successive panes is
given approximately by the following function: $p(h) = 100(0.91)^n$

    (a) What percent of light will pass through 5 panes?
        $p(5) = 2\%$ *plug 5 in for n* $100(0.91)^5$

    (b) What percent of light will pass through 10 panes?
        $p(10) = 1\%$    *plug 10 in for n*    $100(0.91)^{10}$

43) The percentage of patients P who have survived t years after initial diagnosis
of a certain disease is modeled by the function: $P(t) = 100(0.5)^t$

    (a) According to the model, 50% of the patients survive 1 year after initial
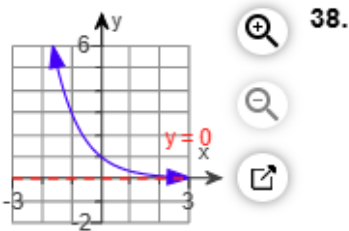    diagnosis. *Plug 1 in for t*

(b) According to the model, 12.5% of the patients survive 3 year after initial diagnosis.  *Plug 3 in for t*

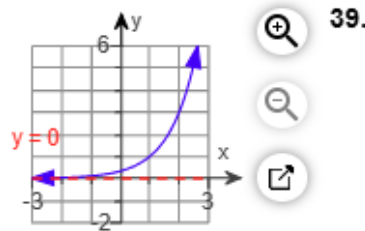(c) As each year passes 50% of the previous year's survivors have survived.

**44)** $D(h) = 9e^{-0.55h}$ can be used to find the number of    milligrams D of a drug that is in a patient's bloodstream h hours after the drug has been administered. How many milligrams will be present after 1 hour? After 11 hours?
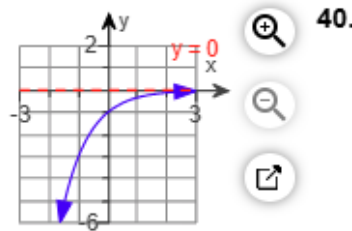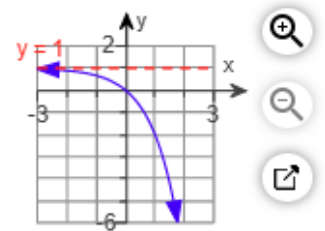
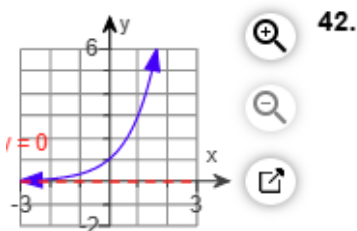$$9e^{-0.55(1)} = \mathbf{5.19}$$    $$9e^{-0.55(11)} = \mathbf{0.021}$$

**38.**



$y = 3^{-x}$

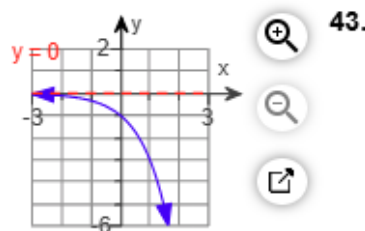**39.**



$y = 3^{x-1}$

**40.**



$y = -3^{-x}$

$y = 1 - 3^{x}$



$y = 3^{x}$

**42.**



$y = -3^{x}$

**43.**



$y = 3^{x} - 1$

**44.**



$y = 3^{1-x}$